

华东交通大学考试专用答题纸 (适用于匿名改卷)

课程: 8.2 P167

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											
阅卷人											

考生注意: 答题时应注明题号, 并按题号顺序答题。

题号	答题内容
P167	理学院 廖国勇
1	设 $z = e^{x^2y}$, 则 $\frac{\partial z}{\partial x} = 2xye^{x^2y}$
2	设 $f(x, y) = x \ln(xy)$, 则 $f'_x(1, e) =$ 解 $f'_x(x, y) = \ln(xy) + x \cdot \frac{1}{xy} \cdot y = \ln(xy) + 1$ $f'_x(1, e) = \ln(1 \cdot e) + 1 = 2$
3	设 $z = \sin(xy)$, 则 $dz =$ 解 $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = y \cos(xy) dx + x \cos(xy) dy$ $= \cos(xy) [y dx + x dy]$
4	设 $z = x^3y - xy^3$, 则 $\frac{\partial^2 z}{\partial x \partial y} =$ 解 $\frac{\partial z}{\partial x} = 3x^2y - y^3$ $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} (\frac{\partial z}{\partial x}) = \frac{\partial}{\partial y} (3x^2y - y^3)$ $= 3x^2 - 3y^2$

题号

P167-P168

答 题 内 容

理学院 廖国勇

二 1 求曲线 $z = \frac{x^2+y^2}{2}$ 在点 $(1,1,1)$ 处的切线对于 x 轴正向所成的

$$y=1$$

倾角 α .

$$\text{解: } \frac{\partial z}{\partial x} = x \quad \frac{\partial z}{\partial x} \Big|_{(1,1)} = 1$$

$$\tan \alpha = \frac{\partial z}{\partial x} \Big|_{(1,1)} = 1 \quad \therefore \alpha = \frac{\pi}{4}$$

2 设 $z = 2\cos^2(x - \frac{y}{2})$, 求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

$$\text{解: } z = 2\cos^2(x - \frac{y}{2}) = \cos(2x - y) + 1$$

$$\frac{\partial z}{\partial x} = -2\sin(2x - y) \quad \frac{\partial z}{\partial y} = -\sin(2x - y) \cdot (-1) = \sin(2x - y)$$

亦可以直接算

$$\frac{\partial z}{\partial x} = 4\cos(x - \frac{y}{2}) \cdot (-\sin(x - \frac{y}{2})) = -2\sin(2x - y)$$

$$\frac{\partial z}{\partial y} = 4\cos(x - \frac{y}{2}) \cdot (-\sin(x - \frac{y}{2})) \cdot (-\frac{1}{2}) = \sin(2x - y)$$

$$\cos 2\theta + 1 = 2\cos^2 \theta$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

3 设 $z = x \ln(x+y)$, 求其二阶偏导数

$$\text{解 } \frac{\partial z}{\partial x} = \ln(x+y) + x \cdot \frac{1}{x+y}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = \frac{1}{x+y} + x \cdot \left(-\frac{1}{(x+y)^2} \right)$$

$$= \frac{x+y}{(x+y)^2} - \frac{x}{(x+y)^2} = \frac{y}{(x+y)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{x+y} + \frac{1}{x+y} + x \cdot \left(-\frac{1}{(x+y)^2} \right) = \frac{x+2y}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = x \cdot \frac{1}{x+y}$$

$$\frac{\partial^2 z}{\partial y^2} = x \cdot \frac{-1}{(x+y)^2} = -\frac{x}{(x+y)^2}$$

题号

P168 - P169

答 题 内 容

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= 4

设 $z = x^3 \sin y - ye^x$, 求 $\frac{\partial^3 z}{\partial x^2 \partial y}$.

$$\text{解: } \frac{\partial z}{\partial x} = 3x^2 \sin y - ye^x \quad \frac{\partial^2 z}{\partial x^2} = 6x \sin y - ye^x$$

$$\frac{\partial^3 z}{\partial x^2 \partial y} = 6x \cos y - e^x$$

5 设 $z = \arctan \frac{y}{x}$, 求 dz

$$\text{解 } \frac{\partial z}{\partial x} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \left(-\frac{y}{x^2}\right)$$

$$= -\frac{y}{x^2 + y^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{1 + \left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2 + y^2}$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = -\frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy$$

6 设 $u = a^{x+yz} - \ln x^a$ ($a > 0$), 求 du

$$\text{解 } \frac{\partial u}{\partial x} = a^{x+yz} \cdot \ln a - \frac{a}{x} \quad \frac{\partial u}{\partial y} = z \cdot a^{x+yz} \cdot \ln a$$

$$\frac{\partial u}{\partial z} = y a^{x+yz} \ln a$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz$$

$$= \left(a^{x+yz} \ln a - \frac{a}{x}\right) dx + z \cdot a^{x+yz} \ln a dy + y a^{x+yz} \ln a dz$$

7 设 $f(t)$ 为连续函数, $u = xyz + \int_{yz}^{xy} f(t) dt$, 求 du

$$\text{解 } \frac{\partial u}{\partial x} = yz + yf(xy)$$

$$\frac{\partial u}{\partial y} = xz + xf(xy) - zf(yz)$$

$$\frac{\partial u}{\partial z} = xy - yf(yz)$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz$$

$$= [yz + yf(xy)] dx + [xz + xf(xy) - zf(yz)] dy + [xy - yf(yz)] dz$$

8 设 $f(x, y, z) = (\frac{x}{y})^z$, 求 $df(1, 1, 1)$

$$\text{解 } \frac{\partial f}{\partial x} = z(\frac{x}{y})^{z-1} \cdot \frac{1}{y} \quad \frac{\partial f}{\partial y} = z(\frac{x}{y})^{z-1} \cdot (-\frac{x}{y^2})$$

$$\frac{\partial f}{\partial z} = (\frac{x}{y})^z \ln(\frac{x}{y})$$

$$\frac{\partial f}{\partial x}|_{(1,1,1)} = 1 \quad \frac{\partial f}{\partial y}|_{(1,1,1)} = -1 \quad \frac{\partial f}{\partial z} = 0$$

$$df(1,1,1) = \frac{\partial f}{\partial x}|_{(1,1,1)} dx + \frac{\partial f}{\partial y}|_{(1,1,1)} dy + \frac{\partial f}{\partial z}|_{(1,1,1)} dz \\ = dx - dy$$

9 讨论函数 $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2 \neq 0 \\ 0, & x^2+y^2 = 0 \end{cases}$ 在点 $(0,0)$ 处的可微性.

$$\text{解 } \frac{\partial f}{\partial x}|_{(0,0)} = \lim_{\Delta x \rightarrow 0} \frac{f(\Delta x, 0) - f(0,0)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{0}{\Delta x} = 0$$

$$\text{同理 } \frac{\partial f}{\partial y}|_{(0,0)} = 0$$

$$\lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{f(0+\Delta x, 0+\Delta y) - \frac{\partial f}{\partial x}|_{(0,0)} \Delta x - \frac{\partial f}{\partial y}|_{(0,0)} \Delta y - f(0,0)}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}$$

$$= \lim_{\substack{\Delta x \rightarrow 0 \\ \Delta y \rightarrow 0}} \frac{\Delta x \cdot \Delta y}{(\Delta x)^2 + (\Delta y)^2} \text{ 不存在}$$

$\therefore$ 函数 $f(x, y)$ 在 $(0,0)$ 处不可微.

三 1 设 $z = \frac{y}{f(x^2-y^2)}$, 其中 $f(u)$ 可导, 证明 $\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{z}{y^2}$

$$\text{证明 } \frac{\partial z}{\partial x} = -\frac{y}{f^2(x^2-y^2)} \cdot 2x \cdot f' = \frac{-2xy}{f^2} \cdot f'$$

$$\frac{\partial z}{\partial y} = \frac{f(x^2-y^2) - y f'(x^2-y^2) \cdot (-2y)}{f^2(x^2-y^2)} = \frac{f(x^2-y^2) + 2y^2 f'(x^2-y^2)}{f^2(x^2-y^2)} = \frac{f + 2yf'}{f^2}$$

$$\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{-2y}{f^2} f' + \frac{f + 2yf'}{y f^2} \\ = \frac{1}{yf} = \frac{1}{y} \cdot \frac{z}{y} = \frac{z}{y^2}$$

题号	答 题 内 容
P170	理学院 廖国勇
三 2	设 $r = \sqrt{x^2 + y^2 + z^2}$, 证明: $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$ 证明: $\frac{\partial r}{\partial x} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \cdot \partial x = \frac{x}{r}$ $\frac{\partial^2 r}{\partial x^2} = \frac{1}{r} + x \cdot \frac{1}{-r^2} \cdot \frac{\partial r}{\partial x} = \frac{1}{r} - \frac{x^2}{r^3}$ 同理 $\frac{\partial^2 r}{\partial y^2} = \frac{1}{r} - \frac{y^2}{r^3}$ $\frac{\partial^2 r}{\partial z^2} = \frac{1}{r} - \frac{z^2}{r^3}$ $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{3}{r} - \frac{x^2 + y^2 + z^2}{r^3} = \frac{3}{r} - \frac{1}{r} = \frac{2}{r}$